



Verbal Reasoning & Quantitative Reasoning Sample Questions with Explanations



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Verbal Reasoning Question Types

The Verbal Reasoning measure contains three types of questions: Reading Comprehension, Text Completion and Sentence Equivalence. [Click here](#) to get a closer look at each, **including sample questions with explanations.**

Reading Comprehension Questions

Introduction

Reading Comprehension questions are designed to test a wide range of abilities that are required in order to read and understand the kinds of prose commonly encountered in graduate school.

Those abilities include:

- understanding the meaning of individual words and sentences
- understanding the meaning of paragraphs and larger bodies of text
- distinguishing between minor and major points
- summarizing a passage
- drawing conclusions from the information provided
- reasoning from incomplete data to infer missing information
- understanding the structure of a text in terms of how the parts relate to one another
- identifying the author's assumptions and perspective
- analyzing a text and reaching conclusions about it
- identifying strengths and weaknesses of a position
- developing and considering alternative explanations

As this list implies, reading and understanding a piece of text requires far more than a passive understanding of the words and sentences it contains; it requires active engagement with the text, asking questions, formulating and evaluating hypotheses and reflecting on the relationship of the particular text to other texts and information.

Typically, about half of the questions on the test are based on passages; each passage has one to six questions associated with it. Most passages are one paragraph long, and one or two are several paragraphs long. Passages are drawn from the physical sciences, biological sciences, social sciences, business, arts and humanities and everyday topics and are based on material found in books and periodicals, both academic and nonacademic.

Questions can cover any of the topics listed above, from the meaning of a particular word to assessing evidence that might support or weaken points made in the passage. Many, but not all, of the questions are standard multiple-choice questions, in which you are required to select a single correct answer; others ask you to select multiple correct answers; and still others ask you to select a sentence from the passage.

Reading Comprehension Question Types

Multiple-choice — Select One Answer Choice

Description

These are traditional multiple-choice questions with five answer choices, of which you must select one.

Tips for Answering

- Read *all* the answer choices before making your selection, even if you think you know the correct answer in advance.
- The correct answer is the one that most accurately and most completely answers the question posed; be careful not to be misled by answer choices that are only partially true or that only partially answer the question. Also, be careful not to pick an answer choice simply because it is a true statement.
- When the question asks about the meaning of a word in the passage, be sure the answer choice you select correctly represents the way the word is being used in the passage. Many words have different meanings when used in different contexts.

Multiple-choice — Select One or More Answer Choices

Description

These questions provide three answer choices and ask you to select all that are correct; one, two or all three of the answer choices may be correct. To gain credit for these questions, you must select all the correct answers, and only those; there is no credit for partially correct answers.

Tips for Answering

- Evaluate each answer choice separately on its own merits; when evaluating one answer choice, do not take the others into account.
- A correct answer choice accurately and completely answers the question posed; be careful not to be misled by answer choices that are only partially true or that only partially answer the question. Also, be careful not to pick an answer choice simply because it is a true statement.
- Do not be disturbed if you think all three answer choices are correct, since questions of this type can have up to three correct answer choices.

Select-in-Passage

Description

These questions ask you to select the sentence in the passage that meets a certain description. To select a sentence, click on any word in the sentence or select the sentence with the keyboard. In longer passages, the question will usually apply to only one or two specified paragraphs; you will not be able to select a sentence elsewhere in the passage.

Note: Because these questions depend on the use of the computer, they do not appear on the paper-delivered, alternate-format test. Equivalent multiple-choice questions are used in their place.

Tips for Answering

- Evaluate each of the relevant sentences in the passage separately before selecting your answer. Do not evaluate any sentences that are outside the paragraphs under consideration.
- A correct answer choice must accurately match the description given in the question; do not select a sentence if any part of the description does not apply to it. However, note that the question need not fully describe all aspects of the sentence.

Reading Comprehension Sample Questions

Questions 1 to 3 are based on this passage.

Reviving the practice of using elements of popular music in classical composition, an approach that had been in hibernation in the United States during the 1960s, composer Philip Glass (born 1937) embraced the ethos of popular music in his compositions. Glass based two symphonies on music by rock musicians David Bowie and Brian Eno, but the symphonies' sound is distinctively his. Popular elements do not appear out of place in Glass's classical music, which from its early days has shared certain harmonies and rhythms with rock music. Yet this use of popular elements has not made Glass a composer of popular music. His music is not a version of popular music packaged to attract classical listeners; it is high art for listeners steeped in rock rather than the classics.

Select only one answer choice.

1. The passage addresses which of the following issues related to Glass's use of popular elements in his classical compositions?
 - A. How it is regarded by listeners who prefer rock to the classics
 - B. How it has affected the commercial success of Glass's music
 - C. Whether it has contributed to a revival of interest among other composers in using popular elements in their compositions
 - D. Whether it has had a detrimental effect on Glass's reputation as a composer of classical music
 - E. Whether it has caused certain of Glass's works to be derivative in quality

Consider each of the three choices separately and select all that apply.

2. The passage suggests that Glass's work displays which of the following qualities?
 - A. A return to the use of popular music in classical compositions
 - B. An attempt to elevate rock music to an artistic status more closely approximating that of classical music
 - C. A long-standing tendency to incorporate elements from two apparently disparate musical styles

3. Select the sentence that distinguishes two ways of integrating rock and classical music.

Explanation

The passage describes in general terms how Philip Glass uses popular music in his classical compositions and explores how Glass can do this without being imitative. Note that there are no opposing views discussed; the author is simply presenting his or her views.

Question 1: One of the important points that the passage makes is that when Glass uses popular elements in his music, the result is very much his own creation (it is “distinctively his”). In other words, the music is far from being derivative. Thus, one issue that the passage addresses is the one referred to in answer choice E — it answers it in the negative. The passage does not discuss the impact of Glass’s use of popular elements on listeners, on the commercial success of his music, on other composers or on Glass’s reputation, so none of Choices A through D is correct. **The correct answer is Choice E.**

Question 2: To answer this question, it is important to assess each answer choice independently. Since the passage says that Glass revived the use of popular music in classical compositions, answer choice A is clearly correct. On the other hand, the passage also denies that Glass composes popular music or packages it in a way to elevate its status, so answer choice B is incorrect. Finally, since Glass’s style has always mixed elements of rock with classical elements, answer choice C is correct. **Thus, the correct answer is Choice A and Choice C.**

Question 3: Almost every sentence in the passage refers to incorporating rock music in classical compositions, but only the last sentence distinguishes two ways of doing so. It distinguishes between writing rock music in a way that will make it attractive to classical listeners and writing classical music that will be attractive to listeners familiar with rock. **Thus, the correct answer is the last sentence of the passage.**

Text Completion Questions

Skilled readers do not simply absorb the information presented on the page; instead, they maintain a constant attitude of interpretation and evaluation, reasoning from what they have read so far to create a picture of the whole and revising that picture as they go. Text Completion questions test this ability by omitting crucial words from short passages and asking the test taker to use the remaining information in the passage as a basis for selecting words or short phrases to fill the blanks and create a coherent, meaningful whole.

Question Structure

- Passage composed of one to five sentences
- One to three blanks
- Three answer choices per blank (five answer choices in the case of a single blank)
- The answer choices for different blanks function independently; i.e., selecting one answer choice for one blank does not affect what answer choices you can select for another blank
- Single correct answer, consisting of one choice for each blank; no credit for partially correct answers

Tips for Answering

Do not merely try to consider each possible combination of answers; doing so will take too long and is open to error. Instead, try to analyze the passage in the following way:

- Read through the passage to get an overall sense of it.
- Identify words or phrases that seem particularly significant, either because they emphasize the structure of the passage (words like *although* or *moreover*) or because they are central to understanding what the passage is about.
- Try to fill in the blanks with words or phrases that seem to complete the sentence, then see if similar words are offered among the answer choices.
- Do not assume that the first blank is the one that should be filled first; perhaps one of the other blanks is easier to fill first. Select your choice for that blank, and then see whether you can complete another blank. If none of the choices for the other blank seem to make sense, go back and reconsider your first selection.
- When you have made your selection for each blank, check to make sure the passage is logically, grammatically and stylistically coherent.

Text Completion Sample Questions

For each blank select one entry from the corresponding column of choices.

Fill all blanks in the way that best completes the text.

1. It is refreshing to read a book about our planet by an author who does not allow facts to be (i)_____ by politics: well aware of the political disputes about the effects of human activities on climate and biodiversity, this author does not permit them to (ii)_____ his comprehensive description of what we know about our biosphere. He emphasizes the enormous gaps in our knowledge, the sparseness of our observations, and the (iii)_____, calling attention to the many aspects of planetary evolution that must be better understood before we can accurately diagnose the condition of our planet.

Sample Question 1 Answers		
Blank (i)	Blank (ii)	Blank (iii)
(A) overshadowed	(D) enhance	(G) plausibility of our hypotheses
(B) invalidated	(E) obscure	(H) certainty of our entitlement
(C) illuminated	(F) underscore	(I) superficiality of our theories

Explanation

The overall tone of the passage is clearly complimentary. To understand what the author of the book is being complimented on, it is useful to focus on the second blank. Here, we must determine what word would indicate something that the author is praised for not permitting. The only answer choice that fits the case is “obscure,” since enhancing and underscoring are generally good things to do, not things one should refrain from doing. Choosing “obscure” clarifies the choice for the first blank; the only choice that fits well with “obscure” is

“overshadowed.” Notice that trying to fill the first blank before filling the second blank is hard — each choice has at least some initial plausibility. Since the third blank requires a phrase that matches “enormous gaps” and “sparseness of our observations,” the best choice is “superficiality of our theories.”

Thus, the correct answer is Choice A (overshadowed), Choice E (obscure) and Choice I (superficiality of our theories).

2. In parts of the Arctic, the land grades into the landfast ice so _____ that you can walk off the coast and not know you are over the hidden sea.

Sample Question 2 Answers

(A) permanently
(B) imperceptibly
(C) irregularly
(D) precariously
(E) relentlessly

Explanation

The word that fills the blank has to characterize how the land grades into the ice in a way that explains how you can walk off the coast and over the sea without knowing it. The word that does that is “imperceptibly” if the land grades imperceptibly into the ice, you might well not know that you had left the land. Describing the shift from land to ice as permanent, irregular, precarious or relentless would not help to explain how you would fail to know.

Thus, the correct answer is Choice B (imperceptibly).

Sentence Equivalence Questions

Like Text Completion questions, Sentence Equivalence questions test the ability to reach a conclusion about how a passage should be completed on the basis of partial information, but to a greater extent they focus on the meaning of the completed whole. Sentence Equivalence questions consist of a single sentence with just one blank, and they ask you to find two choices that lead to a complete, coherent sentence while producing sentences that mean the same thing.

Question Structure

- Consists of:
 - a single sentence
 - one blank
 - six answer choices
- Requires you to select **two** of the answer choices; no credit for partially correct answers

Tips for Answering

Do not simply look among the answer choices for two words that mean the same thing. This can be misleading for two reasons. First, the answer choices may contain pairs of words that mean the same thing but do not fit coherently into the sentence. Second, the pair of words that do constitute the correct answer may not mean exactly the same thing, since all that matters is that the resultant sentences mean the same thing.

- Read the sentence to get an overall sense of it.
- Identify words or phrases that seem particularly significant, either because they emphasize the structure of the sentence (words like *although* or *moreover*) or because they are central to understanding what the sentence is about.
- Try to fill in the blank with a word that seems appropriate to you and then see if two similar words are offered among the answer choices. If you find some word that is similar to what you are expecting but cannot find a second one, do not become fixated on your interpretation; instead, see whether there are other words among the answer choices that can be used to fill the blank coherently.
- When you have selected your pair of answer choices, check to make sure that each one produces a sentence that is logically, grammatically and stylistically coherent, and that the two sentences mean the same thing.

Sentence Equivalence Sample Questions

Select the *two* answer choices that, when used to complete the sentence, fit the meaning of the sentence as a whole *and* produce completed sentences that are alike in meaning.

1. Although it does contain some pioneering ideas, one would hardly characterize the work as _____.
A. orthodox
B. eccentric
C. original
D. trifling
E. conventional
F. innovative

Explanation

The word “Although” is a crucial signpost here. The work contains some pioneering ideas, but apparently it is not overall a pioneering work. Thus, the two words that could fill the blank appropriately are “original” and “innovative.” Note that “orthodox” and “conventional” are two words that are very similar in meaning, but neither one completes the sentence sensibly.

Thus, the correct answer is Choice C (original) and Choice F (innovative).

2. It was her view that the country's problems had been _____ by foreign technocrats, so that to ask for such assistance again would be counterproductive.
- A. ameliorated
 - B. ascertained
 - C. diagnosed
 - D. exacerbated
 - E. overlooked
 - F. worsened

Explanation

The sentence relates a piece of reasoning, as indicated by the presence of "so that": asking for the assistance of foreign technocrats would be counterproductive because of the effects such technocrats have had already. This means that the technocrats must have bad effects; i.e., they must have "exacerbated" or "worsened" the country's problems.

Thus, the correct answer is Choice D (exacerbated) and Choice F (worsened).



Quantitative Reasoning Question Types

The Quantitative Reasoning measure has four types of questions:

- Quantitative Comparison Questions
- Multiple-choice Questions — Select One Answer Choice
- Multiple-choice Questions — Select One or More Answer Choices
- Numeric Entry Questions

[Click here](#) to get a closer look at each, **including sample questions with explanations.**

Each question appears either independently as a discrete question or as part of a set of questions called a [Data Interpretation set](#). All of the questions in a Data Interpretation set are based on the same data presented in tables, graphs or other displays of data.

You can find steps for solving quantitative problems, including useful strategies for answering questions on the Quantitative Reasoning measure, in [Problem-solving Steps](#). In addition, the pages for each of the four Quantitative Reasoning question types and the Data Interpretation set page, mentioned above, contain strategies specific to answering those types of questions.

You are allowed to use a basic calculator on the Quantitative Reasoning measure. For the computer-delivered test, the calculator is provided on-screen. For the paper-delivered test, a handheld calculator is provided at the test center. Read more about [using the calculator](#).

Quantitative Comparison Questions

Description

Questions of this type ask you to compare two quantities — Quantity A and Quantity B — and then determine which of the following statements describes the comparison:

- Quantity A is greater.
- Quantity B is greater.
- The two quantities are equal.
- The relationship cannot be determined from the information given.

Tips for Answering

1. **Become familiar with the answer choices.** Quantitative Comparison questions always have the same answer choices, so get to know them, especially the last choice, “The relationship cannot be determined from the information given.” Never select this last choice if it is clear that the values of the two quantities can be determined by computation. Also, if you determine that one quantity is greater than the other, make sure you carefully select the corresponding choice so as not to reverse the first two choices.
2. **Avoid unnecessary computations.** Don’t waste time performing needless computations in order to compare the two quantities. Simplify, transform or estimate one or both of the given quantities only as much as is necessary to compare them.
3. **Remember that geometric figures are not necessarily drawn to scale.** If any aspect of a given geometric figure is not fully determined, try to redraw the figure, keeping those aspects that are completely determined by the given information fixed but changing the aspects of the figure that are not determined. Examine the results. What variations are possible in the relative lengths of line segments or measures of angles?
4. **Plug in numbers.** If one or both of the quantities are algebraic expressions, you can substitute easy numbers for the variables and compare the resulting quantities in your analysis. Consider all kinds of appropriate numbers before you give an answer: e.g., zero, positive and negative numbers, small and large numbers, fractions and decimals. If you see that Quantity A is greater than Quantity B in one case and Quantity B is greater than Quantity A in another case, choose “The relationship cannot be determined from the information given.”
5. **Simplify the comparison.** If both quantities are algebraic or arithmetic expressions and you cannot easily see a relationship between them, you can try to simplify the comparison. Try a step-by-step simplification that is similar to the steps involved when you solve the equation $5 = 4x + 3$ for x , or similar to the steps involved when you determine that the inequality $\frac{3y+2}{5} < y$ is equivalent to the simpler inequality $1 < y$. Begin by setting up a comparison involving the two quantities, as follows:

$$\text{Quantity A} \quad \boxed{?} \quad \text{Quantity B}$$

where $\boxed{?}$ is a “placeholder” that could represent the relationship *greater than* ($>$), *less than* ($<$) or *equal to* ($=$) or could represent the fact that the relationship cannot be determined from the information given. Then try to simplify the comparison, step-by-step, until you can determine a relationship between simplified quantities. For example, you may conclude after the last step that $\boxed{?}$ represents equal to ($=$). Based on this conclusion, you may be able to compare Quantities A and B. To understand this strategy more fully, see sample questions 6 to 9.

between P and R . Following are two possible variations of Figure 1, each of which is drawn to be consistent with the information $PQ = PR$.

Figure 2

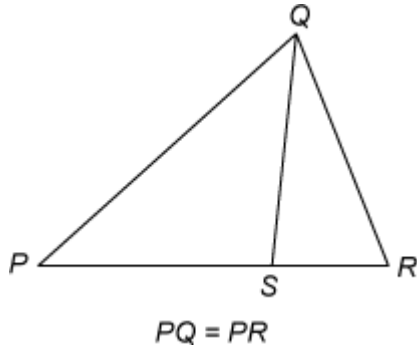
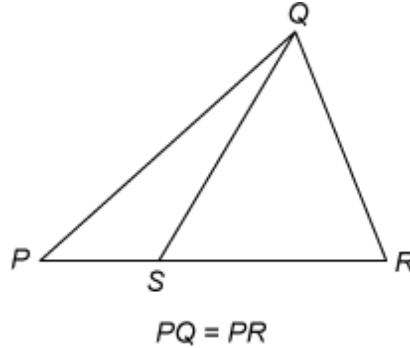


Figure 3



Note that Quantity A is greater in Figure 2 and Quantity B is greater in Figure 3. **Thus, the correct answer is Choice D, the relationship cannot be determined from the information given.**

5. $y = 2x^2 + 7x - 3$

Quantity A

Quantity B

x

y

- A. Quantity A is greater.
- B. Quantity B is greater.
- C. The two quantities are equal.
- D. The relationship cannot be determined from the information given.

Explanation

If $x = 0$, then $y = 2(0^2) + 7(0) - 3 = -3$, so in this case, $x > y$; but

if $x = 1$, then $y = 2(1^2) + 7(1) - 3 = 6$, so in that case, $y > x$. **Thus, the correct answer is Choice D, the relationship cannot be determined from the information given.**

Note that plugging numbers into expressions *may not* be conclusive. It *is* conclusive, however, if you get different results after plugging in different numbers: the conclusion is that the relationship cannot be determined from the information given. It is also conclusive if there are only a small number of possible numbers to plug in and *all* of them yield the same result, say, that Quantity B is greater.

Now suppose there are an infinite number of possible numbers to plug in. If you plug many of them in and each time the result is, for example, that Quantity A is greater, you still cannot conclude that Quantity A is greater for every possible number that could be plugged in. Further analysis would be necessary and should focus on whether Quantity A is greater for all possible numbers or whether there are numbers for which Quantity A is not greater.

The following sample questions focus on simplifying the comparison.

6. $y > 4$

Quantity A

$$\frac{3y+2}{5}$$

Quantity B

$$y$$

- A. Quantity A is greater.
- B. Quantity B is greater.
- C. The two quantities are equal.
- D. The relationship cannot be determined from the information given.

Explanation

Set up the initial comparison:

$$\frac{3y+2}{5} \quad ? \quad y$$

Then simplify:

Step 1: Multiply both sides by 5 to get $3y+2 \quad ? \quad 5y$

Step 2: Subtract $3y$ from both sides to get $2 \quad ? \quad 2y$

Step 3: Divide both sides by 2 to get $1 \quad ? \quad y$

The comparison is now simplified as much as possible. In order to compare 1 and y , note that you are given the information $y > 4$ (above Quantities A and B). It follows from $y > 4$ that $y > 1$, or $1 < y$, so that in the comparison $1 \quad ? \quad y$, the placeholder $?$ represents *less than* $1 < y (<)$:

However, the problem asks for a $1 < y$ comparison between Quantity A and Quantity B, not a comparison between 1 and y . To go from the comparison between 1 and y to a comparison between Quantities A and B, start with the last comparison, $1 < y$, and carefully consider each simplification step in reverse order to determine what each comparison implies about the preceding comparison, all the way back to the comparison between Quantities A and B if possible. Since Step 3 was “*divide* both sides by 2,” *multiplying* both sides of the comparison $1 < y$ by 2 implies the preceding comparison $2 < 2y$, thus reversing Step 3. Each simplification step can be reversed as follows:

- A. Reverse Step 3: *multiply* both sides by 2.
- B. Reverse Step 2: *add* $3y$ to both sides.
- C. Reverse Step 1: *divide* both sides by 5.

When each step is reversed, the relationship remains *less than* (<), so Quantity A is less than Quantity B. **Thus, the correct answer is Choice B, Quantity B is greater.**

While some simplification steps like subtracting 3 from both sides or dividing both sides by 10 are always reversible, it is important to note that some steps, like squaring both sides, may not be reversible.

Also, note that when you simplify an *inequality*, the steps of multiplying or dividing both sides by a negative number change the direction of the inequality; for example, if $x < y$, then $-x > -y$. relationship in the final, simplified inequality may be the *opposite* of the relationship between Quantities A and B. This is another reason to consider the impact of each step carefully.

- | | | | |
|----|-----------------------------|--|-------------------|
| 7. | <u>Quantity A</u> | | <u>Quantity B</u> |
| | $\frac{2^{30} - 2^{29}}{2}$ | | 2^{28} |
- A. Quantity A is greater.
 B. Quantity B is greater.
 C. The two quantities are equal.
 D. The relationship cannot be determined from the information given.

Explanation

Set up the initial comparison:

$$\frac{2^{30} - 2^{29}}{2} \quad ? \quad 2^{28}$$

Then simplify:

Step 1: Multiply both sides by 2 to get $2^{30} - 2^{29} \quad ? \quad 2^{29}$

Step 2: Add 2^{29} to both sides to get $2^{30} \quad ? \quad 2^{29} + 2^{29}$

Step 3: Simplify the right-hand side using the fact that $(2)(2^{29}) = 2^{30}$ to get $2^{30} \quad ? \quad 2^{30}$

The resulting relationship is *equal to* (=). In reverse order, each simplification step implies *equal to* in the preceding comparison. So Quantities A and B are also equal. **Thus, the correct answer is Choice C, the two quantities are equal.**

8. Quantity A Quantity B
- $x^2 + 1$ $2x - 1$
- A. Quantity A is greater.
 - B. Quantity B is greater.
 - C. The two quantities are equal.
 - D. The relationship cannot be determined from the information given.

Explanation

Set up the initial comparison:

$$x^2 + 1 \quad ? \quad 2x - 1$$

Then simplify by noting that the quadratic polynomial $x^2 - 2x + 1$ can be factored:

Step 1: Subtract $2x$ from both sides to get $x^2 - 2x + 1 \quad ? \quad -1$

Step 2: Factor the left-hand side to get $(x - 1)^2 \quad ? \quad -1$

The left-hand side of the comparison is the square of a number. Since the square of a number is always greater than or equal to 0, and 0 is greater than -1 , the simplified comparison is the inequality $(x - 1)^2 > -1$ and the resulting relationship is *greater than* ($>$). In reverse order, each simplification step implies the inequality *greater than* ($>$) in the preceding comparison. Therefore, Quantity A is greater than Quantity B. **The correct answer is Choice A, Quantity A is greater.**

9.

$$w > 1$$

Quantity A

$$7w - 4$$

Quantity B

$$2w + 5$$

- A. Quantity A is greater.
- B. Quantity B is greater.
- C. The two quantities are equal.
- D. The relationship cannot be determined from the information given.

Explanation

Set up the initial comparison:

$$7w - 4 \quad ? \quad 2w + 5$$

Then simplify:

Step 1: Subtract $2w$ from both sides and add 4 to both sides to get $5w \quad ? \quad 9$

Step 2: Divide both sides by 5 to get $w \quad ? \quad \frac{9}{5}$

The comparison cannot be simplified any further. Although you are given that $w > 1$, you still don't know how w compares to $\frac{9}{5}$, or 1.8. For example, if $w = 1.5$, then $w < 1.8$, but if $w = 2$, then $w > 1.8$. In other words, the relationship between w and $\frac{9}{5}$ cannot be determined. Note that each of these simplification steps is reversible, so in reverse order, each simplification step implies that the *relationship cannot be determined* in the preceding comparison. Thus, the relationship between Quantities A and B cannot be determined. **The correct answer is Choice D, the relationship cannot be determined from the information given.**

The strategy of simplifying the comparison works most efficiently when you note that a simplification step is reversible while actually taking the step. Here are some common steps that are always reversible:

- Adding any number or expression to both sides of a comparison
- Subtracting any number or expression from both sides
- Multiplying both sides by any nonzero number or expression
- Dividing both sides by any nonzero number or expression

Remember that if the relationship is an *inequality*, multiplying or dividing both sides by any *negative* number or expression will yield the *opposite* inequality. Be aware that some common operations like squaring both sides are generally not reversible and may require

further analysis using other information given in the question in order to justify reversing such steps.

Multiple-choice Questions — Select One Answer Choice

These questions are multiple-choice questions that ask you to select only one answer choice from a list of five choices.

Tips for Answering

1. **Use the fact that the answer is there.** If your answer is not one of the five answer choices given, you should assume that your answer is incorrect and do the following:
 - a. Reread the question carefully — you may have missed an important detail or misinterpreted some information.
 - b. Check your computations — you may have made a mistake, such as mis-keying a number on the calculator.
 - c. Reevaluate your solution method — you may have a flaw in your reasoning.
2. **Examine the answer choices.** In some questions you are asked explicitly which of the choices has a certain property. You may have to consider each choice separately or you may be able to see a relationship between the choices that will help you find the answer more quickly. In other questions, it may be helpful to work backward from the choices, say, by substituting the choices in an equation or inequality to see which one works. However, be careful, as that method may take more time than using reasoning.
3. **For questions that require approximations, scan the answer choices to see how close an approximation is needed.** In other questions, too, it may be helpful to scan the choices briefly before solving the problem to get a better sense of what the question is asking. If computations are involved in the solution, it may be necessary to carry out all computations exactly and round only your final answer in order to get the required degree of accuracy.

In other questions, you may find that estimation is sufficient and will help you avoid spending time on long computations.

Multiple-choice Questions — Select One Answer Choice Sample Questions

Select a single answer choice.

1. If $5x + 32 = 4 - 2x$, what is the value of x ?

- A. -4
- B. -3
- C. 4
- D. 7
- E. 12

Explanation

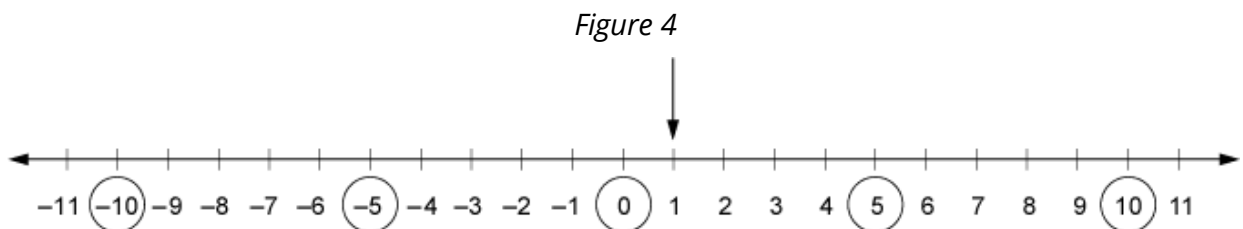
Solving the equation for x , you get $7x = -28$, and so $x = -4$. **The correct answer is Choice A, -4.**

2. Which of the following numbers is farthest from the number 1 on the number line?

- A. -10
- B. -5
- C. 0
- D. 5
- E. 10

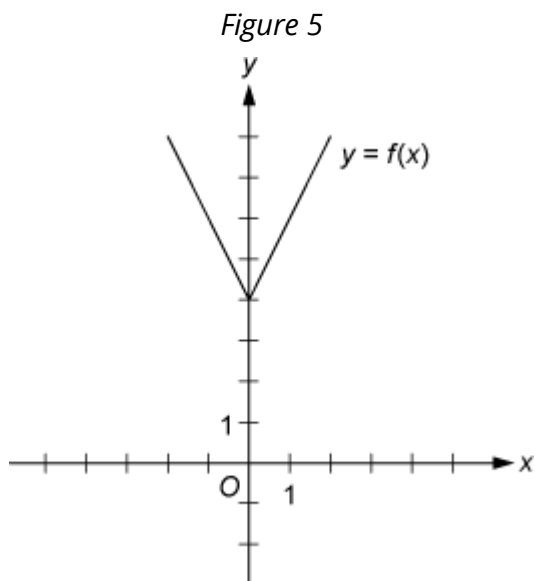
Explanation

Circling each of the answer choices in a sketch of the number line (Figure 4) shows that of the given numbers, -10 is the greatest distance from 1.



Another way to answer the question is to remember that the distance between two numbers on the number line is equal to the absolute value of the difference of the two numbers. For example, the distance between -10 and 1 is $|-10-1|=11$, and the distance between 10 and 1 is $|10-1|=|9|=9$. **The correct answer is Choice A, -10 .**

3.



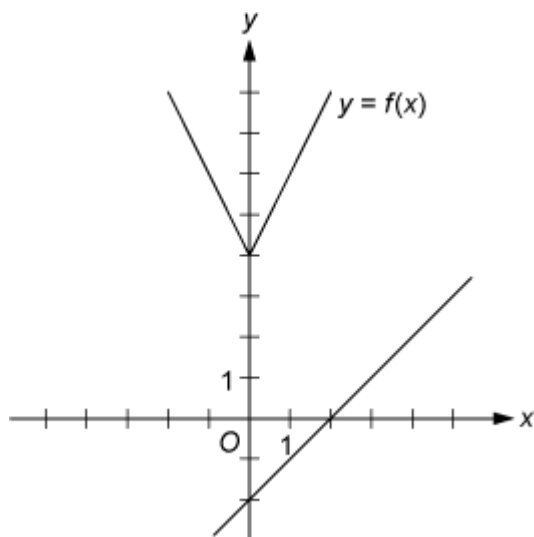
The figure above shows the graph of the function f , defined by $f(x) = |2x| + 4$ for all numbers x . For which of the following functions g , defined for all numbers x , does the graph of g intersect the graph of f ?

- A. $g(x) = x - 2$
- B. $g(x) = x + 3$
- C. $g(x) = 2x - 2$
- D. $g(x) = 2x + 3$
- E. $g(x) = 3x - 2$

Explanation

You can see that all five choices are linear functions whose graphs are lines with various slopes and y -intercepts. The graph of Choice A is a line with slope 1 and y -intercept -2 , shown in Figure 6.

Figure 6



It is clear that this line will not intersect the graph of f to the left of the y -axis. To the right of the y -axis, the graph of f is a line with slope 2, which is greater than slope 1. Consequently, as the value of x increases, the value of y increases faster for f than for g , and therefore the graphs do not intersect to the right of the y -axis. Choice B is similarly ruled out. Note that if the y -intercept of either of the lines in Choices A and B were greater than or equal to 4 instead of less than 4, they would intersect the graph of f .

Choices C and D are lines with slope 2 and y -intercepts less than 4. Hence, they are parallel to the graph of f (to the right of the y -axis) and therefore will not intersect it. Any line with a slope greater than 2 and a y -intercept less than 4, like the line in Choice E, will intersect the graph of f (to the right of the y -axis). The correct answer is Choice E, $g(x) = 3x - 2$.

4. A car got 33 miles per gallon using gasoline that cost \$2.95 per gallon. Approximately what was the cost, in dollars, of the gasoline used in driving the car 350 miles?
- A. \$10
 - B. \$20
 - C. \$30
 - D. \$40
 - E. \$50

Explanation

Scanning the answer choices indicates that you can do at least some estimation and still answer confidently. The car used $\frac{350}{33}$ gallons of gasoline, so the cost was $\left(\frac{350}{33}\right)(2.95)$ dollars.

You can estimate the product $\left(\frac{350}{33}\right)(2.95)$ by estimating $\frac{350}{33}$ a little low, 10, and estimating 2.95 a little high, 3, to get approximately $(10)(3) = 30$ dollars. You can also use the calculator to compute a more exact answer and then round the answer to the nearest 10 dollars, as suggested by the answer choices. The calculator yields the decimal $31.287\dots$, which rounds to 30 dollars. **Thus, the correct answer is Choice C, \$30.**

5. A certain jar contains 60 jelly beans — 22 white, 18 green, 11 yellow, 5 red, and 4 purple. If a jelly bean is to be chosen at random, what is the probability that the jelly bean will be neither red nor purple?
- A. 0.09
 - B. 0.15
 - C. 0.54
 - D. 0.85
 - E. 0.91

Explanation

Since there are 5 red and 4 purple jelly beans in the jar, there are 51 that are neither red nor purple and the probability of selecting one of these is $\frac{51}{60}$. Since all of the answer choices are decimals, you must convert the fraction to its decimal equivalent, 0.85. **Thus, the correct answer is Choice D, 0.85.**

Multiple-choice Questions — Select One or More Answer Choices

Description

These questions are multiple-choice questions that ask you to select one or more answer choices from a list of choices. A question may or may not specify the number of choices to select.

Tips for Answering

1. **Note whether you are asked to indicate a specific number of answer choices or all choices that apply.** In the latter case, be sure to consider all of the choices, determine which ones are correct, and select all of those and only those choices. Note that there may be only one correct choice.
2. **In some questions that involve conditions that limit the possible values of the numerical answer choices, it may be efficient to determine the least and/or the greatest possible value.** Knowing the least and/or greatest possible value may enable you to quickly determine all of the choices that are correct.
3. **Avoid lengthy calculations by recognizing and continuing numerical patterns.**

Multiple-choice Questions — Select One or More Answer Choices Sample Questions

Select one or more answer choices according to the specific question directions.

If the question does not specify how many answer choices to select, select all that apply.

- The correct answer may be just one of the choices or as many as all of the choices, depending on the question.
- No credit is given unless you select all of the correct choices and no others.

If the question specifies how many answer choices to select, select exactly that number of choices.

1. Which two of the following numbers have a product that is between -1 and 0 ?

Indicate both of the numbers.

- A. -20
- B. -10
- C. 2^{-4}
- D. 3^{-2}

Explanation

For this question, you must select a pair of answer choices. The product of the pair must be negative, so the possible products are $(-20)(2^{-4})$, $(-20)(3^{-2})$, $(-10)(2^{-4})$ and $(-10)(3^{-2})$. The product must also be greater than -1 . The first product $\frac{-20}{2^4} = -\frac{20}{16} < -1$, the second product is $\frac{-20}{3^2} = -\frac{20}{9} < -1$ and the third product is $\frac{-10}{2^4} = -\frac{10}{16} > -1$ so you can stop there. **The correct answer consists of Choices B (-10) and C (2^{-4}).**

2. Which of the following integers are multiples of both 2 and 3?

Indicate all such integers.

- A. 8
- B. 9
- C. 12
- D. 18
- E. 21
- F. 36

Explanation

You can first identify the multiples of 2, which are 8, 12, 18 and 36, and then among the multiples of 2 identify the multiples of 3, which are 12, 18 and 36. Alternatively, if you realize that every number that is a multiple of 2 and 3 is also a multiple of 6, you can identify the choices that are multiples of 6. **The correct answer consists of Choices C (12), D (18) and F (36).**

3. Each employee of a certain company is in either Department X or Department Y, and there are more than twice as many employees in Department X as in Department Y. The average (arithmetic mean) salary is \$25,000 for the employees in Department X and \$35,000 for the employees in Department Y. Which of the following amounts could be the average salary for all of the employees of the company?

Indicate all such amounts.

- A. \$26,000
- B. \$28,000
- C. \$29,000
- D. \$30,000
- E. \$31,000
- F. \$32,000
- G. \$34,000

Explanation

One strategy for answering this kind of question is to find the least and/or greatest possible value. Clearly the average salary is between \$25,000 and \$35,000, and all of the answer choices are in this interval. Since you are told that there are more employees with the lower average salary, the average salary of all employees must be less than the average of \$25,000 and \$35,000, which is \$30,000. If there were exactly twice as many employees in Department X as in Department Y, then the average salary for all employees would be, to the nearest dollar, the following weighted mean,

$$\frac{(2)(25,000) + (1)(35,000)}{2 + 1} \approx 28,333 \text{ dollars}$$

where the weight for \$25,000 is 2 and the weight for \$35,000 is 1. Since there are more than twice as many employees in Department X as in Department Y, the actual average salary must be even closer to \$25,000 because the weight for \$25,000 is greater than 2. This means that \$28,333 is the greatest possible average. Among the choices given, the possible values of the average are therefore \$26,000 and \$28,000. **Thus, the correct answer consists of Choices A (\$26,000) and B (\$28,000).**

Intuitively, you might expect that any amount between \$25,000 and \$28,333 is a possible value of the average salary. To see that \$26,000 is possible, in the weighted mean above, use the respective weights 9 and 1 instead of 2 and 1. To see that \$28,000 is possible, use the respective weights 7 and 3.

4. Which of the following could be the units digit of 57^n , where n is a positive integer?

Indicate all such digits.

- A. 0
- B. 1
- C. 2
- D. 3
- E. 4
- F. 5
- G. 6
- H. 7
- I. 8
- J. 9

Explanation

The units digit of 57^n is the same as the units digit of 7^n for all positive integers n . To see why this is true for $n = 2$, compute 57^2 by hand and observe how its units digit results from the units digit of 7^2 . Because this is true for every positive integer n , you need to consider only powers of 7. Beginning with $n = 1$ and proceeding consecutively, the units digits of $7, 7^2, 7^3, 7^4$ and 7^5 are 7, 9, 3, 1 and 7, respectively. In this sequence, the first digit, 7, appears again, and the pattern of four digits, 7, 9, 3, 1, repeats without end. Hence, these four digits are the only possible units digits of 7^n and therefore of 57^n . **The correct answer consists of Choices B (1), D (3), H (7) and J (9).**

Numeric Entry Questions

Description

Questions of this type ask you either to enter your answer as an integer or a decimal in a single answer box or to enter it as a fraction in two separate boxes — one for the numerator and one for the denominator. You will use the computer mouse and keyboard to enter your answer.

Tips for Answering

1. **Make sure you answer the question that is asked.** Since there are no answer choices to guide you, read the question carefully and make sure you provide the type of answer required. Sometimes there will be labels before or after the answer box to indicate the appropriate type of answer. Pay special attention to units such as feet or miles, to orders of magnitude such as millions or billions, and to percents as compared with decimals.
2. **If you are asked to round your answer, make sure you round to the required degree of accuracy.** For example, if an answer of 46.7 is to be rounded to the nearest integer, you need to enter the number 47. If your solution strategy involves intermediate computations, you should carry out all computations exactly and round only your final answer in order to get the required degree of accuracy. If no rounding instructions are given, enter the exact answer.
3. **Examine your answer to see if it is reasonable with respect to the information given.** You may want to use estimation or another solution path to double-check your answer.

Numeric Entry Sample Questions

Enter your answer as an integer or a decimal if there is a single answer box OR as a fraction if there are two separate answer boxes — one for the numerator and one for the denominator.

To enter an integer or a decimal, either type the number in the answer box using the keyboard or use the Transfer Display button on the calculator.

- First, select the answer box — a cursor will appear in the box — and then type the number.
- For a negative sign, type a hyphen. For a decimal point, type a period.
- The Transfer Display button on the calculator will transfer the calculator display to the answer box.
- Equivalent forms of the correct answer, such as 2.5 and 2.50, are all correct.
- Enter the exact answer unless the question asks you to round your answer.

To enter a fraction, type the numerator and the denominator in their respective answer boxes using the keyboard.

- Select each answer box — a cursor will appear in the box — then type an integer. A decimal point cannot be used in either box.
- For a negative sign, type a hyphen in either box.
- The Transfer Display button on the calculator cannot be used for a fraction.

- Fractions do **not** need to be reduced to lowest terms, though you may need to reduce your fraction to fit in the boxes.

- One pen costs \$0.25 and one marker costs \$0.35. At those prices, what is the total cost of 18 pens and 100 markers?

\$

Explanation

Multiplying \$0.25 by 18 yields \$4.50, which is the cost of the 18 pens, and multiplying \$0.35 by 100 yields \$35.00, which is the cost of the 100 markers. The total cost is therefore $\$4.50 + \$35.00 = \$39.50$. Equivalent decimals, such as \$39.5 or \$39.500, are considered correct.

Thus, the correct answer is \$39.50 (or equivalent).

Note that the dollar symbol is in front of the answer box, so the symbol \$ does not need to be entered in the box. In fact, only numbers, a decimal point and a negative sign can be entered in the answer box.

- Rectangle R has length 30 and width 10, and square S has length 5. The perimeter of S is what fraction of the perimeter of R ?

Explanation

The perimeter of R is $30 + 10 + 30 + 10 = 80$, and the perimeter of S is $(4)(5) = 20$. Therefore, the perimeter of S is $\frac{20}{80}$ of the perimeter of R . To enter the answer $\frac{20}{80}$, you should enter the numerator 20 in the top box and the denominator 80 in the bottom box. Because the fraction does not need to be reduced to lowest terms, any fraction that is equivalent to $\frac{20}{80}$ is also considered correct, as long as it fits in the boxes. For example, both of the fractions $\frac{2}{8}$ and $\frac{1}{4}$ are considered correct. **Thus, the correct answer is $\frac{20}{80}$ (or any equivalent fraction).**

3.

Figure 7
Results of a Used-Car Auction

	Small Cars	Large Cars
Number of cars offered	32	23
Number of cars sold	16	20
Projected sales total for cars offered (in thousands)	\$70	\$150
Actual sales total (in thousands)	\$41	\$120

For the large cars sold at an auction that is summarized in the table above, what was the average sale price per car?

\$

Explanation

From Figure 7, you see that the number of large cars sold was 20 and the sales total for large cars was \$120,000 (not \$120). Thus, the average sale price per car was $\frac{\$120,000}{20} = \$6,000$.

The correct answer is \$6,000 (or equivalent).

(In numbers that are 1,000 or greater, you do not need to enter commas in the answer box.)

4. A merchant made a profit of \$5 on the sale of a sweater that cost the merchant \$15. What is the profit expressed as a percent of the merchant's cost?

Give your answer to the nearest whole percent.

%

Explanation

The percent profit is $\left(\frac{5}{15}\right)(100) = 33.333\dots = 33.\bar{3}$ percent, which is 33%, to the nearest whole percent. **Thus, the correct answer is 33% (or equivalent).**

If you use the calculator and the Transfer Display button, the number that will be transferred to the answer box is 33.333333, which is incorrect since it is not given to the nearest whole percent. You will need to adjust the number in the answer box by deleting all of the digits to the right of the decimal point.

Also, since you are asked to give the answer as a percent, the decimal equivalent of 33%, which is 0.33, is incorrect. The percent symbol next to the answer box indicates that the form of the answer must be a percent. Entering 0.33 in the box would give the erroneous answer 0.33%.

5. Working alone at its constant rate, machine A produces k liters of a chemical in 10 minutes. Working alone at its constant rate, machine B produces k liters of the chemical in 15 minutes. How many minutes does it take machines A and B , working simultaneously at their respective constant rates, to produce k liters of the chemical?

minutes

Explanation

Machine A produces $\frac{k}{10}$ liters per minute, and machine B produces $\frac{k}{15}$ liters per minute. So, when the machines work simultaneously, the rate at which the chemical is produced is the sum

of these two rates, which is $\frac{k}{10} + \frac{k}{15} = k\left(\frac{1}{10} + \frac{1}{15}\right) = k\left(\frac{25}{150}\right) = \frac{k}{6}$ liters per minute. To compute the time required to produce k liters at this rate, divide the amount k by the rate $\frac{k}{6}$ to

get $\frac{k}{\frac{k}{6}} = 6$.

Therefore, the correct answer is 6 minutes (or equivalent).

One way to check that the answer of 6 minutes is reasonable is to observe that if the slower rate of machine B were the same as machine A 's faster rate of k liters in 10 minutes, then the two machines, working simultaneously, would take half the time, or 5 minutes, to produce the k liters. So the answer has to be *greater than 5 minutes*. Similarly, if the faster rate of machine A were the same as machine B 's slower rate of k liters in 15 minutes, then the two machines would take half the time, or 7.5 minutes, to produce the k liters. So the answer has to be *less than 7.5 minutes*. Thus, the answer of 6 minutes is reasonable compared to the lower estimate of 5 minutes and the upper estimate of 7.5 minutes.

Data Interpretation Sets

Description

Data Interpretation questions are grouped together and refer to the same table, graph or other data presentation. These questions ask you to interpret or analyze the given data. The types of questions may be Multiple-choice (both types) or Numeric Entry.

Tips for Answering

1. **Scan the data presentation briefly to see what it is about, but do not spend time studying all of the information in detail.** Focus on those aspects of the data that are necessary to answer the questions. Pay attention to the axes and scales of graphs; to the units of measurement or orders of magnitude (such as *billions*) that are given in the titles, labels and legends; and to any notes that clarify the data.
2. **When graphical data presentations, such as bar graphs and line graphs, are shown with scales, you should read, estimate or compare quantities by sight or by measurement, according to the corresponding scales.** For example, you can use the relative sizes of bars or sectors to compare the quantities that they represent, but be aware of broken scales and of bars that do not start at 0.
3. **The questions are to be answered only on the basis of the data presented, everyday facts (such as the number of days in a year) and your knowledge of mathematics.** Do not make use of specialized information you may recall from other sources about the particular context on which the questions are based unless the information can be derived from the data presented.

Data Interpretation Sample Questions

Questions 1 to 3 are based on the following data.

Figure 8

Annual Percent Change in Dollar Amount of Sales at Five Retail Stores from 2006 to 2008

Store	Percent Change from 2006 to 2007	Percent Change from 2007 to 2008
P	10	-10
Q	-20	9
R	5	12
S	-7	-15
T	17	-8

1. If the dollar amount of sales at Store *P* was \$800,000 for 2006, what was the dollar amount of sales at that store for 2008?
- A. \$727,200
 - B. \$792,000
 - C. \$800,000
 - D. \$880,000
 - E. \$968,000

Explanation

According to Figure 8, if the dollar amount of sales at Store *P* was \$800,000 for 2006, then it was 10% greater for 2007, which is 110% of that amount, or \$880,000. For 2008 the amount was 90% of \$880,000, which is \$792,000. **The correct answer is Choice B, \$792,000.**

Note that an increase of 10% for one year and a decrease of 10% for the following year does not result in the same dollar amount as the original dollar amount because the base that is used in computing the percents is \$800,000 for the first change but \$880,000 for the second change.

2. At Store *T*, the dollar amount of sales for 2007 was what percent of the dollar amount of sales for 2008?

Give your answer to the nearest 0.1%.

 %

Explanation

If *A* is the dollar amount of sales at Store *T* for 2007, then 8% of *A*, or $0.08A$, is the amount of decrease from 2007 to 2008. Thus $A - 0.08A = 0.92A$ is the dollar amount for 2008.

Therefore, the desired percent can be obtained by dividing *A* by $0.92A$, which equals $\frac{A}{0.92A} = \frac{1}{0.92} = 1.0869565\dots$. Expressed as a percent and rounded to the nearest 0.1% this number is 108.7%. **Thus, the correct answer is 108.7% (or equivalent).**

3. Based on the information given, which of the following statements must be true?

Indicate all such statements.

- A. For 2008 the dollar amount of sales at Store *R* was greater than that at each of the other four stores.
- B. The dollar amount of sales at Store *S* for 2008 was 22% less than that for 2006.
- C. The dollar amount of sales at Store *R* for 2008 was more than 17% greater than that for 2006.

Explanation

For Choice A, since the only data given in Figure 8 are percent changes from year to year, there is no way to compare the actual dollar amount of sales at the stores for 2008 or for any other year. Even though Store *R* had the greatest percent increase from 2006 to 2008, its actual dollar amount of sales for 2008 may have been much smaller than that for any of the other four stores, and therefore Choice A is not necessarily true.

For Choice B, even though the sum of the 2% decreases would suggest a 22% decrease, the bases of the percents are different. If B is the dollar amount of sales at Store *S* for 2006, then the dollar amount for 2007 is 93% of B , or $0.93B$, and the dollar amount for 2008 is given by $(0.85)(0.93)B$, which is $0.7905B$. Note that this represents a percent decrease of $100 - 79.05 = 20.95\%$, which is not equal to 22%, and so Choice B is not true.

For Choice C, if C is the dollar amount of sales at Store *R* for 2006, then the dollar amount for 2007 is given by $(1.12)(1.05)C$, which is $1.176C$. Note that this represents a 17.6% increase, which is greater than 17%, so Choice C must be true.

Therefore, the correct answer consists of only Choice C (The dollar amount of sales at Store *R* for 2008 was more than 17% greater than that for 2006).

